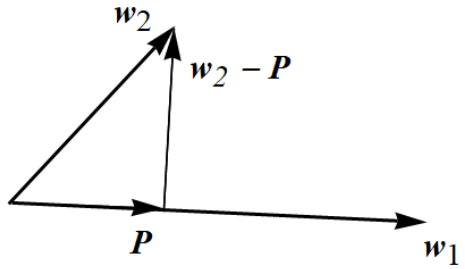
Exercise 6

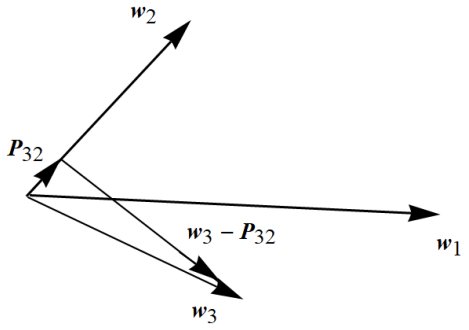
Illustrate the Gram–Schmidt orthogonality method by sketching two linearly independent vectors w_1 and w_2 in the plane that are not orthogonal. Then do it with three vectors in space.

Solution

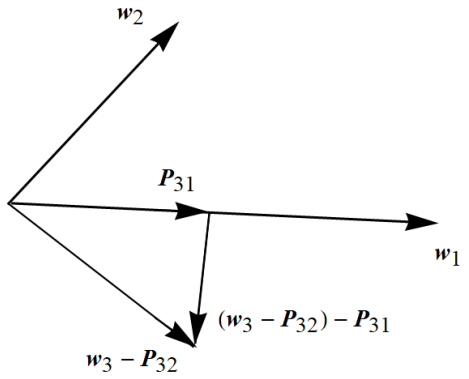
Suppose there are two linearly independent vectors, $\mathbf{w}_1$ and $\mathbf{w}_2$. The idea of the Gram–Schmidt procedure is to subtract from $\mathbf{w}_2$ the component parallel to $\mathbf{w}_1$ in order to obtain a vector perpendicular to $\mathbf{w}_1$.



Let there be a third linearly independent vector $\mathbf{w}_3$. To obtain a vector perpendicular to both $\mathbf{w}_1$ and $\mathbf{w}_2$, subtract from $\mathbf{w}_3$ the components parallel to $\mathbf{w}_1$ and $\mathbf{w}_2$ in succession.



The vector $\mathbf{w}_3 - \mathbf{P}_{32}$ is perpendicular to $\mathbf{w}_2$ but not to $\mathbf{w}_1$.



The vector $\mathbf{w}_3 - \mathbf{P}_{32} - \mathbf{P}_{31}$ is perpendicular to both $\mathbf{w}_1$ and $\mathbf{w}_2$.